IntroAstro 2012-2013

Homework Set 1-A Solutions

1. The resolution of a telescope is the smallest angular distance between objects it can distinuish. As seen by the telescope, on Earth, the two sides of the crater must be separated by at least 2" to be distinguishable – *resolved*. We can draw a triangle whose base is the crater's diameter and whose apex is the telescope on Earth. The triangle has two sides of essentially equal, and large, length, and one very short one, opposite the very small angle at the apex. The **small-angle formula** relates the length of the short side AB to the angle α and the length of the long side R as

$$\frac{\mathrm{AB}}{R} = \frac{\alpha}{57.3^\circ} = \frac{\alpha}{206265"} \ .$$

In our case we know $R = 3.844 \times 10^5$ km and $\alpha = 2"$ so we can solve for AB. Since we are given α in arc-seconds it is easiest to use the second form of the equation to write

$$\mathrm{AB} = \frac{\alpha}{206265"} R = \frac{2"}{206265"} \cdot 3.84 \times 10^5 \,\mathrm{km} = 3.727244 \,\mathrm{km} \ .$$

Since we had R in km the answer came out in km. Rounding this to two significant digits as requested I find

$$\mathrm{AB} = 3.7 \,\mathrm{km} \ .$$

I would enter this as **3.7** since we do not include units in numerical responses.

2. We can do this calculation by first finding the angle *subtended* by the Moon, i.e. the angle formed at Earth between lines connecting to the two ends of a diameter of the Moon. We can then use the distance to the Sun and the angle we found to determine the radius of the Sun. A shorter way, with less computation, less room for error, and sometimes even more insight, is to use *scaling*. If we call the angle subtended by both Sun and Moon α, then the diameters $2R_{☾}, 2R_{\odot}$ of Moon and Sun (written in terms of their radii), and their respective distances from Earth $D_{☾}, D_{\odot}$ satisfy

$$\frac{2R_{☾}}{D_{☾}} = \frac{\alpha}{206265"}$$
$$\frac{2R_{\odot}}{D_{\odot}} = \frac{\alpha}{206265"} \ .$$

We can sidestep the need to find α by noting the two expressions on the left-hand side (LHS) are equal:

$$\frac{2R_{\leftmoon}}{D_{\leftmoon}} = \frac{2R_{\odot}}{D_{\odot}} \ .$$

Cancelling the common factor and rearranging, we have

$$\frac{R_{\odot}}{R_{\leftmoon}} = \frac{D_{\odot}}{D_{\leftmoon}} \ .$$

This has all been a possibly long-winded way to find this result, which simply states that since the two relevant triangles are *similar* – because they have the same angle at their apex (and are both effectively isosceles triangles) – their corresponding sides are in the same proportion. But we will use this type of argument many times in cases where the result is not so obvious, so I am taking this opportunity to introduce it. After all that the solution is anticlimactic. Multiply by $R_{\leftmoon}$ to find

$$R_{\odot} = \frac{D_{\odot}}{D_{\leftmoon}} R_{\leftmoon} \ .$$

Before plugging numbers in, check that both sides have the same units. Indeed, the fraction on the RHS is dimensionless - a ratio of two distances. So we can safely compute - taking care to express both distances in the same units so the ratio of numbers is indeed the ratio of distances:

$$R_{\odot} = \frac{149.6 \times 10^6}{3.844 \times 10^5} \cdot 1737\,\mathrm{km} = 676002.1\,\mathrm{km} \ .$$

Rounding this to three significant digits I obtain 676000 km, or if you prefer (I do) 6.76×10^5 km. I enter this into the Coursera quiz system as **6.76e5**.

3. Geosynchronous orbit in general can refer to any orbit with a period equal to a sidereal day. The special geosynchronous orbit I describe here, in which the satellite remains above a fixed point somewhere on Earth's equator, is more specifically a *geostationary* orbit. The satellite's position on the celestial sphere then coincides with the Zenith of an observer at this point. It thus lies along the celestial equator.

   Looking at this from the point of view of a stationary observer out in space, we see this: As the Earth spins to the East, the satellite moves to the East as well, remaining at the rotating Zenith. Its RA is precisely the *sidereal time* at the point above which it lies. Thus the RA increases, completing a revolution of the celestial equator to the East each sidereal day.

Looking at the same situaiton from the point of view of an observer at the special point on the equator, the satellite appears not to move at all - it is "parked" at the Zenith. Since stars - and the celestial sphere - appear to rotate from East to West, this means that the satellite is not stationary *on the celestial sphere* but moves from West to East along the sphere at just the right rate to cancel the motion of the sphere.

The answer to this one is **B**.

Note there are a few subtleties being ignored here. The satellite is not *very* far from Earth. As we will see, it is about $6R_\oplus$ from the Earth's center ($R_\oplus$ is the Earth's radius). This is near enough that observers far enough North will see the satellite to the *South* of the celestial equator, and conversely obserevers in the South will see it *North* of the equator. For the observer at the equator, what we said is precisely true.

4. Any star crosses the meridian when sidereal time is equal to the star's RA. Thus Sirius will cross the meridian at ST 6:45 and Rigel at ST 5:15. Neglecting the small difference (can you figure it out? Should find about 10 seconds) between a sidereal hour and a Solar hour, we see that Rigel will cross the meridian about 1.5 hours before Sirius. Note that where we observe is irrelevant, so long as we can see these stars.

   The answer is **A**.

5. When a star crosses the meridian, its *Zenith angle* Z is given by the (absolute value of the) difference between its declination and the latitude of the observer. Sirius has declination $-16°$ while Auckland lies at latitude $-37°$. Sirius is *North* of the Zenith (at a higher declination) by 21°. In equations, if you prefer, letting L stand for latitude and δ for declination, we have

$$Z = |\delta - L| = 21° \ .$$

   Since the Zenith is at altitude 90°, the point lying 21° directly North of it is at Altitude $90 - 21 = 69°$. In equations, altitude A and Zenith angle Z are related by

$$A + Z = 90° \ .$$

   North of the Zenith, Sirius is at Azimuth 0°.

   The answer here is **A**.

If you prefer to "see" things rather than use equations, simnply recall that in Auckland the Zenith lies at declination $-37°$. At meridian crossing the Zenith and the star lie on the same meridian, but at different declinations. The angular distance between them is the difference in declinations.

At this time, Sirius will be *at* the Zenith for observers at longitude 175° East and latitude $-16°$, which would put them some 250 km Northwest of Fiji